NASA TN D-437

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TECHNICAL NOTE
D-437

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ON A ONCE-A-DAY BASIS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON November 1960

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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A TWO-IMPULSE PLAN FOR PERFORMING RENDEZVOUS ON A ONCE-A-DAY BASIS

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SUMMARY

An investigation of a two-impulse plan for performing rendezvous on a once-a-day basis with a near-earth satellite station indicates that launch into rendezvous from slightly less than maximum satellite latitude is an unusually favorable circumstance in that no appreciable expense in mass ratio is incurred. In addition, it was found for the two-impulse maneuver employed in this study that the optimum angular travel of the ferry vehicle to rendezvous was considerably less than the 180° transfer which is optimum for the two-impulse in-plane launch.

INTRODUCTION

An important consideration in the operation of a manned space station is the convenience with which materials and personnel can be transported to and from the station. From the standpoint of efficiency it is desirable to use trajectories for launch to rendezvous that are in the orbital plane of the satellite station. However, these in-plane trajectories do not insure once-a-day capability because the period of an efficient launch trajectory is about the same as that of a near-earth satellite; hence, the satellite must be approximately overhead when launch to rendezvous is made. That is to say because of the fact that the period of the useful launch trajectories and the period of the satellite orbit are about the same, there is little compensating ability on the part of the rendezvous vehicle with regard to trip time. If the launch point is allowed to move slightly out of the orbital plane of the satellite as the earth rotates in order to gain time for the satellite to move into a position adjacent to the launch point, then once-a-day rendezvous capability can be insured for a direct launch to rendezvous. Rendezvous may be accomplished by this procedure for any orbital altitude.

The conditions associated with various approaches to orbital rendezvous and the significance of the condition of adjacency of the launch point and satellite when launch to rendezvous is made is discussed in references 1, 2, and 3. There are other approaches to the rendezvous problem, of course, such as the use of long period, highly elliptic orbits and time delay or parking orbits, but these all tend to place more severe requirements on the guidance system for long periods of time and are not considered here.

The purpose of the present investigation was to study the relative cost in mass ratio required to insure once-a-day rendezvous. In the plan studied, once-a-day capability is obtained as mentioned previously by permitting the launch point to be carried out of the orbital plane
of the satellite by the earth's rotation in an amount sufficient to L
permit the satellite to move around its orbit into a position adjacent 1
to the launch point. The exact positioning at adjacency is chosen as 1
that which permits an efficient launch to rendezvous. When a satisfac- 3
tory condition of launch is achieved the ferry vehicle is launched to 8
meet the orbiting station at the ferry vehicle apogee.

At this point the ferry vehicle trajectory is corrected to coincide to the orbit of the station by appropriate application of thrust. This application of thrust serves the dual purpose of bringing the ferry vehicle to orbital speed and correcting its direction of flight to account for the out-of-plane launch. (See fig. 1.) The investigation of reference 4 utilizes this boost to orbital speed and correction of flight-path direction at apogee for considerations of launch into equatorial orbits from northerly latitudes.

In this investigation a simplified analysis of this method of rendezvous was made by utilizing the Keplerian orbital relationships with impulsive applications of thrust at launch and apogee. An assessment of the penalty in mass ratio relative to an in-plane launch was made for various degrees of out-of-plane launching. The degree of out-of-plane launching required to insure once-a-day rendezvous capability was determined for various orbital inclinations and launch-point latitudes. The analysis is concerned with near-earth rendezvous with space stations moving in circular orbits although the technique employed applies to the general case of elliptic orbits.

SYMBOLS

e base of natural logarithms

g_0 acceleration of gravity at surface of earth, 32.2 ft/sec^2

h_a apogee altitude of rendezvous vehicle, nautical miles

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I_{sp} specific impulse, sec

l_a acute angle between path of rendezvous vehicle at rendezvous and path of satellite, deg

l_s inclination of plane of satellite orbit with respect to plane of equator, deg

l_0 launch-point latitude, deg

MR mass ratio, ratio of initial mass to final mass of rendezvous vehicle

MR_{opt} minimum mass ratio that will effect a successful rendezvous from a launch point having a given offset

r_a distance from center of earth to rendezvous point, ft

r_0 radius of earth, 20.91×10^6, ft

t time lapse between intersection of launch point with orbital plane and instant of firing of rendezvous vehicle, min

V_a velocity of rendezvous vehicle at apogee of its trajectory, ft/sec

V_{s_a} velocity of a satellite in circular orbit at the apogee altitude of rendezvous vehicle, ft/sec

ΔV_1 impulsive velocity supplied to rendezvous vehicle at launch, ft/sec

ΔV_2 impulsive velocity supplied to rendezvous vehicle at apogee of its trajectory, ft/sec

α angle through which earth rotates while satellite moves into proper position for launching rendezvous vehicle, measured in plane of launch-point latitude and from the time at which the launch point is in plane of satellite orbit, deg

β minor arc of great circle through intersection of launch point with plane of satellite orbit and location of launch point at time the rendezvous vehicle is launched, deg

γ_0 angle with respect to local horizontal at which rendezvous vehicle is launched, deg

δ_a angular distance rendezvous vehicle travels to apogee (that is, rendezvous), measured about center of earth and in plane of vehicle's trajectory, deg

ϵ eccentricity of elliptic trajectory of rendezvous vehicle

η offset, great-circle distance from launch point at time of launch of rendezvous vehicle perpendicular to trace of satellite orbit on surface of earth, deg

$\dot{\theta}_e$ rate of rotation of earth, 0.250684 deg/min

$\dot{\theta}_s$ rate of rotation of satellite about earth, deg/min

ξ,μ,κ spherical angles used to derive offset, deg

ANALYTICAL PROCEDURE

General Plan

The procedure used to evaluate the cost in mass ratio required to insure once-a-day rendezvous capability was first to determine the amount that the launch point would be offset η from the orbital plane under the worst launch condition to be faced. This condition was taken to be the condition for which the satellite was 180° away from the launch point when the orbital plane was intercepted. This condition necessitates that the launch be delayed one-half an orbital period for the satellite to travel to a condition of adjacency to the launch point where an efficient launch to rendezvous may be made. Under this condition the earth rotates an appreciable amount and causes an offset η or a great circle distance to exist between the orbital plane and the launch point. This offset requires an orbital-plane correction for the ferry vehicle at apogee.

The next step was to assess the expense of launch to rendezvous for various amounts of offset up to the maximum value that can occur as previously defined. This was done in terms of the initial and final velocity increments and the resulting mass ratio. Finally, the results of the calculations of required offset and mass ratio for various offsets were combined to obtain the required cost to insure once-a-day rendezvous capability. This cost was assessed relative to the expense of an in-plane launch to rendezvous having the same angular travel to apogee as the offset launch.

Assumptions and Approximations

It was assumed in this analysis that the largest amount of offset would be encountered for the situation in which the satellite is 180° away from the launch point at the time that the launch point intercepts the plane of the satellite orbit. In addition, it was assumed that the offset angle that defines the required launch condition for rendezvous is determined by the time required for the satellite to travel this 180° in its orbit. That is to say, it was assumed that the offset is defined by the necessity to wait for one-half the satellite period after orbital plane intercept before launch is made. Actually, the satellite must move through 180° and, in addition, an amount proportional to the earth's rotation which occurs during the waiting period. This additional effect is small because of the large difference in the satellite and earth rotational periods and is neglected here.

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The offset angle is less for launches made one-half a satellite period early than for launches made one-half a satellite period late for southerly launches from northerly latitudes because of the nonlinearity of the problem. (See appendix A.) Nevertheless, for simplicity and because the result is conservative, the criterion of one-half a satellite period of wait was used to define the offset angle for this analysis. This consideration is most in error, percentagewise, for launch points near the maximum satellite latitude, but very little offset is required for once-a-day rendezvous capability in these cases.

The effect of the earth's rotation on the initial impulse ΔV_1 was neglected in this analysis. Whereas including the earth's rotation would appreciably affect the impulsive-velocity and mass-ratio curves, the effect becomes small when the final assessment of cost is made relative to the in-plane launch. Sample calculations made to test this assumption indicated that neglecting the earth's rotation had only a small effect on the final answers.

Determination of Required Offset

The offset angle or great circle distance of the launch point from the plane of the station orbit in terms of earth rotation, launch-point latitude, station orbital inclination, and launch delay time is

$$\sin \eta = \sin \alpha \left[\sin l_0 \cos l_s \tan \frac{1}{2} \alpha + \left(\cos^2 l_0 - \cos^2 l_s \right)^{1/2} \right]$$

where

$$\alpha = \dot{\theta}_e t$$

$$t = \frac{360}{2} \frac{1}{\dot{\theta}_s}$$

The geometry concerned with the quantities in this expression is given in figure 2, and a brief derivation of this expression and of other pertinent expressions of the analysis is given in appendix A. The term t is the time required for the satellite to travel 180° in its orbit and hence is the maximum delay required for a once-a-day rendezvous. This expression for η gives the offset angle corresponding to a given period of wait after the satellite plane is intercepted. It has no significance for launch locations at latitudes greater than the maximum satellite latitude in that no intercept occurs with the orbital plane in such cases. This formula determines the offset associated with either one of the two possible intercepts of the orbital plane. In order to simplify the calculations, only one of the two possible intercepts was considered in this analysis. In effect, only southerly launches from northerly latitudes were considered.

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Determination of Velocity Impulses

As mentioned previously rendezvous was considered to be accomplished in two impulses, one at launch from the ground, and one at the apogee of the resulting trajectory. The latter impulse served to turn the ferry vehicle into the plane of the satellite orbit and, in addition, to bring the ferry vehicle to satellite speed. All launches to rendezvous were designed to rise to apogee at the height of the satellite station. No effort was made to overshoot the station in altitude in order to gain time and hence effect rendezvous at smaller offset angles. The maneuver considered is shown in figure 1. The expressions for calculation of the required initial and final velocity impulses were obtained from the Keplerian orbital relationships and were arranged to have the initial flight-path angle γ_0, offset angle η, and the orbital altitude r_a, as independent variables. These relationships are:

$$\Delta V_1 = V_{s_a}\left[\frac{\left(1 + \epsilon^2\right) - 2\epsilon \cos \delta_a}{1 - \epsilon}\right]^{1/2}$$

$$\Delta V_2 = \left[\left(V_{s_a} - V_a \cos l_a\right)^2 + \left(V_a \sin l_a\right)^2\right]^{1/2}$$

$$V_a = V_{s_a}(1 - \epsilon)^{1/2}$$

$$V_{s_a} = \frac{r_0 g_0^{1/2}}{r_a^{1/2}}$$

$$l_a = \sin^{-1}\left(\frac{\sin \eta}{\sin \delta_a}\right)$$

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$$\epsilon = \frac{\left(\frac{r_a}{r_0}\right)^2 - 2\left(\frac{r_a}{r_0}\right)\cos^2\gamma_0 + \cos^2\gamma_0}{\left(\frac{r_a}{r_0}\right)^2 - \cos^2\gamma_0}$$

$$\delta_a = 180 - \tan^{-1}\left[\frac{r_a(1 - \epsilon)\tan \gamma_0}{r_a(1 - \epsilon) - r_0}\right]$$

Determination of Mass Ratios

The required mass ratios for launch to rendezvous were determined on the basis of the two impulsive thrust applications discussed previously. A mean specific impulse of 235 seconds was used in the calculations. This value was considered to be the mean specific impulse of the fuel and associated tankage and nozzles. The expression required to calculate the mass ratio from the previously determined impulses ΔV_1 and ΔV_2 is

$$MR = e^{\frac{\Delta V_1 + \Delta V_2}{g_0 I_{sp}}}$$

RESULTS AND DISCUSSION

Presentation of Results

The maximum value of offset η that will ever be required for assuring rendezvous once a day for firing in a southerly direction from northerly latitudes is shown in figure 3 as a function of the ratio of launch-point latitude to the maximum satellite latitude for orbital inclinations of 0°, 15°, 30°, 45°, 60°, and 90° and satellite altitudes of 400, 500, and 600 nautical miles. Offset angle is shown in figure 4 as a function of time for a launch site located at the maximum satellite latitude. Results are given for various satellite orbital inclinations. The velocity increments at launch and apogee necessary to effect rendezvous are shown as a function of angular distance to apogee in figure 5. Results are given for offsets of 0°, 1°, 2°, 4°, and 10° for satellite altitudes of 400, 500, and 600 nautical miles. The total velocity increment required for rendezvous is shown in figure 6 for these same conditions. The mass ratio required for rendezvous is shown as a function of angular distance to apogee in figure 7 for various offsets and satellite altitudes. These results were obtained for a specific impulse of 235 seconds. The minimum values of the curves of figure 7 are shown as a function of the offset of the launch point from the orbital plane in figure 8 for the three satellite altitudes considered. The required cost of once-a-day rendezvous is shown as function of the ratio of launch-point latitude to maximum satellite latitude in figure 9 and in table I. The cost is given as the percent increase in mass ratio above that required for an inplane launch. Results are given for various satellite orbital inclinations and altitudes.

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Required Offset Angles

The minimum required offset of the launch point from the satellite orbital plane that is necessary to insure once-a-day rendezvous capability occurs when the launch-point latitude is near the maximum satellite latitude. (See fig. 3.) At this latitude the launch point moves in a path that is nearly tangent to the orbital plane of the satellite with which rendezvous is to be made. For this reason there is a substantial period of time when the offset is small. This latter point is illustrated in figure 4. The period of a 400-nautical-mile circular satellite orbit is about 99 minutes. During this period of time $\left(\pm 49\frac{1}{2}\text{ minutes}\right)$, there is a very small offset between the launch point and orbital plane. For a 45° inclined orbit, the maximum offset in the 99-minute period never exceeds 0.68°. (See fig. 4.) Reference 2 points out the advantage of this tangency condition from a slightly different point of view.

Although not included in the analysis made for this report, it can be shown that the offsets indicated by any one curve of figure 4 may be reduced by one-half by simply considering the launch point to be located south of the maximum satellite latitude a distance equal to one-half the maximum offset indicated by the appropriate curve of figure 4. This effect is not shown in figure 3 because of the relatively small abscissa distance in which this effect occurs. The expressions used to evaluate the curves of figure 3 do not include this consideration. (See appendix A.)

The maximum required offsets occur for launches from equatorial sites into polar orbits. About $12\frac{1}{2}^{\circ}$ offset is required for launch from the equator into a polar orbit at an altitude of 400 nautical miles. Relatively small offsets are required for launches into polar orbits for sites near the pole. About $4\frac{3}{4}^{\circ}$ offset is required for launch into a 400-nautical-mile polar orbit from the arctic circle. The required offsets for once-a-day rendezvous increase slightly with orbital altitude in the range investigated.

Required Velocity Impulses

The optimum distance of travel along the earth's surface to rendezvous with respect to minimum total velocity addition is 180° for an inplane launch. (See fig. 6.) However, when offset of the launch point from the orbital plane exists, the optimum trip within the limits defined in this analysis is appreciably less than 180°. (See fig. 6.) For increasing offset from 0° to 10°, the optimum-length trip moves rapidly from 180° to about 70°. For offsets from 1° to 4°, the optimum trip is about 90° in length. The minimum is very shallow, however, and little penalty in total velocity is incurred as a result of using trips substantially longer or shorter than the optimum. An offset of as little as 1° moves the optimum trip from 180° to about 100°.

The optimum-length trip is regulated in substantial measure by the expense of the boost at apogee. (See fig. 5.) For very slow or very long trips this boost at apogee which is employed to change direction and to increase speed becomes large. This effect can be seen in figure 5 which shows the required velocity increments at launch and at apogee. It is readily evident that offsets of 1° to 2° carry only minor penalties in velocity increment, but that offsets of 10° are expensive in the required velocity increments for rendezvous.

Required Mass Ratios

The mass ratios required for rendezvous for various amounts of offset of the launch point from the orbital plane (fig. 7) show the same optimum travel to rendezvous as do the curves of required velocity increment. The mass ratios for rendezvous are not increased greatly by offsets of 1° to 2° but are affected substantially by offsets near 10°. For an offset of 10° an increase of about 50 percent in mass ratio relative to the inplane launch is required for launch to rendezvous with a 400-nautical-mile satellite. The optimum mass ratio for rendezvous increases with rendezvous altitude. (See fig. 8.) This effect is most pronounced at small offsets. A smaller effect of altitude exists at offsets near 10°. The rate of increase of required mass ratio for rendezvous with increase in offset for the small offset range is shown in figure 8.

Cost of Rendezvous

The required cost of once-a-day rendezvous was assessed by using the required offset angles of figure 3 and the required mass ratios for rendezvous with various offsets of the launch point from figure 8. This assessment was made relative to the expense of an inplane launch of the same length of travel as the optimum offset trip. The cost of rendezvous is considered to be the percent increase in mass ratio required to accomplish the offset rendezvous over and above the mass ratio required for the inplane launch. This cost of rendezvous is directly associated with the necessity to change the orbital plane at apogee.

It is apparent from figure 9 that the cost for once-a-day rendezvous is small for launch from near the maximum satellite latitude. Table I gives the required cost in mass ratio to insure once-a-day rendezvous capability for launches into orbits of various inclinations from the maximum satellite latitude. These quantities are too small for satisfactory representation in figure 9. The required cost associated with launching from a point slightly south of the maximum satellite latitude in order to cut the required offset in half is also shown in table I. The cost associated with all of these launches is seen to be less than 1 percent.

The cost associated with launches from the equator into inclined orbits is substantial. Launch into rendezvous with a 400-nautical-mile polar orbit from the equator requires an increase in mass ratio of 65 percent relative to an inplane launch. Launches into polar orbits from regions relatively near the poles is fairly inexpensive in mass ratio. A launch to rendezvous at 400 nautical miles from the arctic circle may be accomplished at a cost of 14 percent in mass ratio. The arctic circle crosses the upper part of Alaska. Launch to rendezvous with a 400-nautical-mile orbit having the inclination of the Mercury Project orbit $\left(\text{maximum latitude of } 32\frac{1}{2}^{\circ}\right)$ may be made once-a-day from Cape Canaveral $\left(\text{latitude } 28\frac{1}{2}^{\circ}\right)$ at a required cost of about 8 percent in mass ratio.

CONCLUDING REMARKS

An investigation of a two-impulse plan for performing rendezvous on a once-a-day basis with a near-earth satellite station indicates that launch into rendezvous from a latitude slightly less than the maximum satellite latitude is a favorable circumstance in that only a small expense in mass ratio is incurred. In addition, it was found for the two-impulse maneuver employed in this study that the optimum angular travel of the ferry vehicle to rendezvous was considerably less than the 180° transfer which is optimum for the two-impulse inplane launch. The optimum angular travel for useful offset angles is from 70° to 120°.

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Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., July 18, 1960.

APPENDIX A

DERIVATION OF OFFSET, RENDEZVOUS ANGLE, AND IMPULSIVE VELOCITY REQUIRED AT RENDEZVOUS

Offset

For the purpose of this derivation, the offset η is defined as the great circle distance from the launch point to the trace of the satellite orbit on the surface of the earth at some time t before or after the launch point intersects the trace of the satellite orbit and is measured perpendicular to the trace. In addition, it should be noted that for the spherical earth analysis followed here, the maximum latitude attained by the satellite orbit is equal to the inclination of the satellite orbit with respect to the equatorial plane. Therefore, it is possible to develop the expression for offset resulting from time delay for a launch point located at some northern latitude utilizing only a quadrant of the northern hemisphere.

Figure 2 illustrates the situation in which the satellite orbit is traced on a quadrant of the northern hemisphere. The launch point is also shown rotating with the earth at some northern latitude less than the maximum satellite latitude. The minor arc of the great circle between the intercept of the plane of the satellite orbit and the plane of the launch-point latitude and the location of the launch point at time of launch of the rendezvous vehicle is shown with its attendant angle at the center of the earth β.

The lead or delay angle α may be written as a function of lead or delay time t thusly:

$$\alpha = \dot{\theta}_e t$$

where $\dot{\theta}_e$ is the rate of rotation of the earth about its axis and t is the lead or delay time (that is, time before or after the launch point is in the plane of the satellite orbit).

The chord subtended at the launch-point latitude l_0 by the angle α is also subtended by the angle β at the center of the earth. The relationship between α and β is given by the expression:

$$\sin \frac{1}{2} \beta = \sin \frac{1}{2} \alpha \cos l_0 \qquad (1)$$

From figure 2, by utilizing the relationships of spherical trigonometry, the following expression may be written for offset due to delay time:

$$\sin \eta = \sin \beta \sin \xi \tag{2}$$

Also from figure 2 it can be seen that:

$$\xi + \mu + \kappa = 180^\circ$$

thus,

$$\sin \xi = \sin\left[180^\circ - (\mu + \kappa)\right] = \sin(\mu + \kappa)$$

and

$$\sin \xi = \sin \mu \cos \kappa + \cos \mu \sin \kappa \tag{3}$$

From spherical trigonometry the functions of μ and κ may be written as follows:

$$\sin \kappa = \frac{\sin\left(90^\circ - l_s\right)}{\sin\left(90^\circ - l_0\right)} = \frac{\cos l_s}{\cos l_0} \tag{4a}$$

$$\cos \kappa = \left(1 - \sin^2\kappa\right)^{1/2} = \frac{\left(\cos^2 l_0 - \cos^2 l_s\right)^{1/2}}{\cos l_0} \tag{4b}$$

$$\cos \mu = \frac{\tan \frac{1}{2} \beta}{\tan\left(90^\circ - l_0\right)} = \tan \frac{1}{2} \beta \tan l_0 \tag{4c}$$

$$\sin \mu = \left(1 - \cos^2\mu\right)^{1/2} = \left(1 - \tan^2 \frac{1}{2} \beta \tan^2 l_0\right)^{1/2} \tag{4d}$$

From the construction of figure 2 the values for ξ, μ, and κ are seen to fall within the range 0° to 90° for values of l_0 from 0° to 90° and values of l_s from 0° to 90°. Therefore, the sine and cosine functions are positive. Substituting equations (3) and (4) into equation (2) gives:

$$\sin \eta = \sin \beta \left[\left(1 - \tan^2 \frac{1}{2} \beta \tan^2 l_0\right)^{1/2} \frac{\left(\cos^2 l_0 - \cos^2 l_s\right)^{1/2}}{\cos l_0} + \frac{\tan \frac{1}{2} \beta \tan l_0 \cos l_s}{\cos l_0} \right]$$

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Expanding the resulting equation yields:

$$\sin \eta = \frac{2 \sin \frac{1}{2} \beta \cos \frac{1}{2} \beta}{\cos \frac{1}{2} \beta \cos^2 l_0} \left[\left(\cos^2 \frac{1}{2} \beta \cos^2 l_0 - \sin^2 \frac{1}{2} \beta \sin^2 l_0\right)^{1/2} \left(\cos^2 l_0 - \cos^2 l_s\right)^{1/2} + \sin \frac{1}{2} \beta \sin l_0 \cos l_s \right]$$

Substituting the relationship of equation (1) into this equation yields:

$$\sin \eta = \frac{2 \sin \frac{1}{2} \alpha}{\cos l_0} \left\{ \left[\cos^2 l_0 - \cos^2 l_0 \sin^2 \frac{1}{2} \alpha \left(\cos^2 l_0 + \sin^2 l_0\right)\right]^{1/2} \left(\cos^2 l_0 - \cos^2 l_s\right)^{1/2} + \sin \frac{1}{2} \alpha \cos l_0 \sin l_0 \cos l_s \right\}$$

Reducing the resulting equation yields:

$$\sin \eta = 2 \sin \frac{1}{2} \alpha \cos \frac{1}{2} \alpha \left[\left(\cos^2 l_0 - \cos^2 l_s\right)^{1/2} + \sin l_0 \cos l_s \tan \frac{1}{2} \alpha \right]$$

thus

$$\sin \eta = \sin \alpha \left[\left(\cos^2 l_0 - \cos^2 l_s\right)^{1/2} + \sin l_0 \cos l_s \tan \frac{1}{2} \alpha \right] \tag{5}$$

Similarly it can be shown that the offset as a function of time before the launch point intersects the trace of the satellite orbit is given by the expression:

$$\sin \eta = \sin \alpha \left[\left(\cos^2 l_0 - \cos^2 l_s \right)^{1/2} - \sin l_0 \cos l_s \tan \frac{1}{2} \alpha \right] \tag{6}$$

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Equations (5) and (6), as derived, give the offset resulting, respectively, from launch after and before the launch point crosses the trace of the satellite orbit with the trace moving from its maximum latitude to the equator. Likewise equations (6) and (5) give the offset resulting, respectively, from launch after and before the launch point crosses the trace of the satellite orbit with the trace moving from the equator to its maximum latitude.

Rendezvous Angle

Since the rendezvous angle l_a, is an acute angle of a right spherical triangle with the offset η the side opposite the rendezvous angle and the angular (arc) distance to rendezvous δ_a the side opposite the right angle, the rendezvous angle can be written as a function of η and δ_a.

$$\sin l_a = \frac{\sin \eta}{\sin \delta_a}$$

Impulsive Velocity Required at Rendezvous

Finally, it can be seen from the following sketch that the impulsive velocity ΔV_2 required to bring the rendezvous vehicle up to circular satellite velocity V_{s_a} and at the same time to rotate its velocity V_a so as to coincide with that of the satellite may be found from the expression:

$$\Delta V_2 = \left[\left(V_{s_a} - V_a \cos l_a \right)^2 + \left(V_a \sin l_a \right)^2 \right]^{1/2}$$

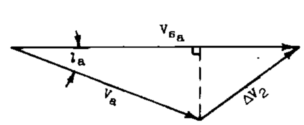

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TABLE I

MAXIMUM COST OF LAUNCH TO RENDEZVOUS FROM NEAR

MAXIMUM SATELLITE LATITUDE

Satellite orbit inclination, deg	Launch site at maximum satellite latitude		Launch site located for minimum offset		
	Maximum offset, deg	Cost, percent	Launch latitude / Maximum satellite latitude	Maximum offset, deg	Cost, percent
400-nautical-mile satellite					
0	0	0	1.00	0	0
15	.34	.35	.989	.17	.18
30	.58	.62	.990	.29	.30
45	.68	.75	.992	.34	.35
60	.58	.62	.995	.29	.30
90	0	0	1.000	0	0
500-nautical-mile satellite					
0	0	0	1.000	0	0
15	.36	.34	.988	.18	.17
30	.63	.60	.990	.32	.32
45	.73	.72	.992	.37	.35
60	.63	.60	.995	.32	.32
90	0	0	1.000	0	0
600-nautical-mile satellite					
0	0	0	1.000	0	0
15	.39	.28	.987	.20	.15
30	.68	.50	.989	.34	.25
45	.79	.82	.991	.40	.30
60	.68	.50	.994	.34	.25
90	0	0	1.000	0	0

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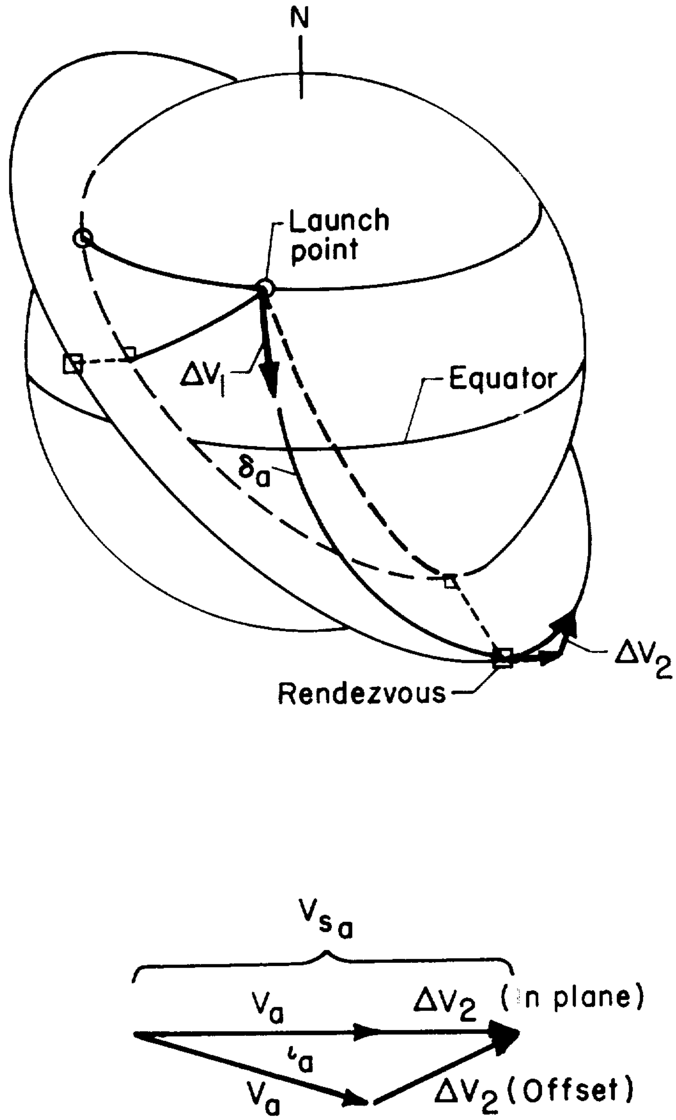

Figure 1.- Direct two-impulse rendezvous with launch-point offset.

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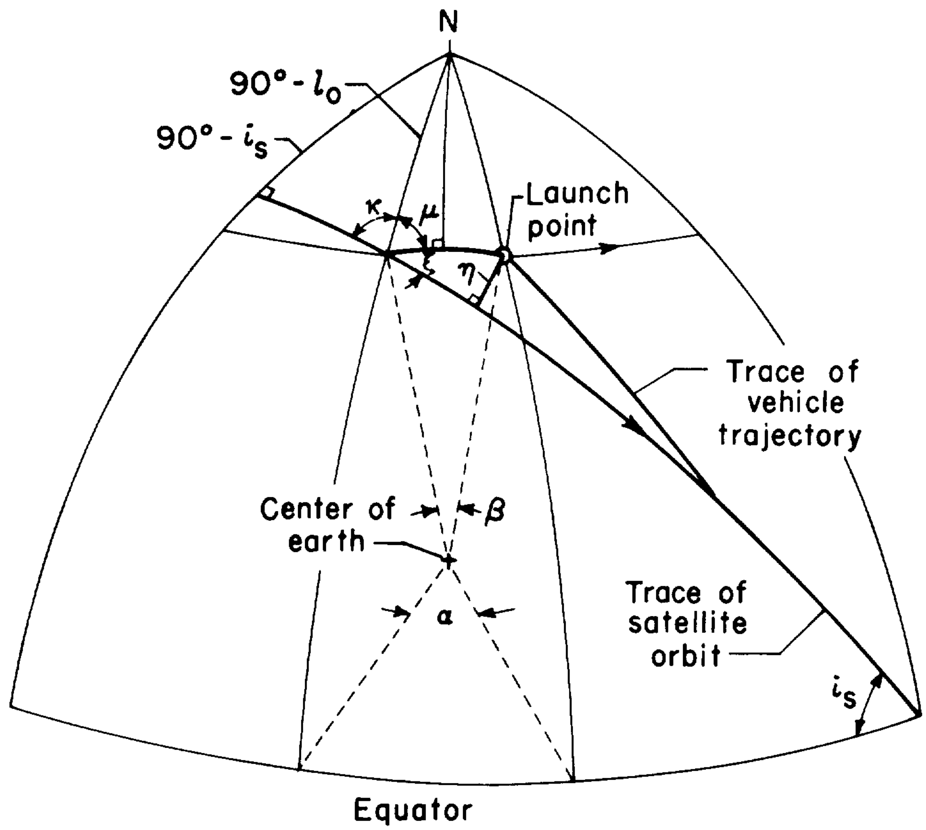
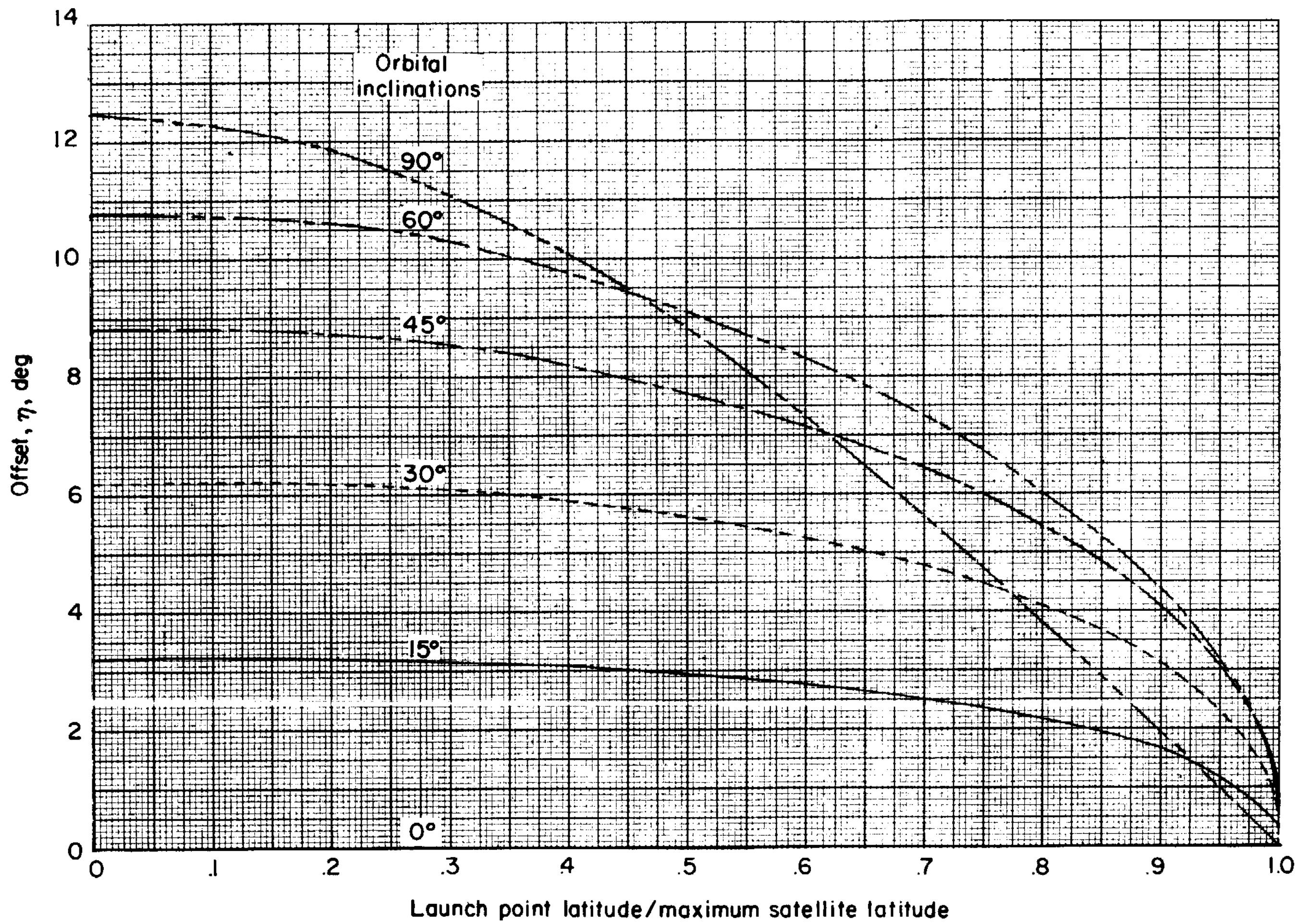
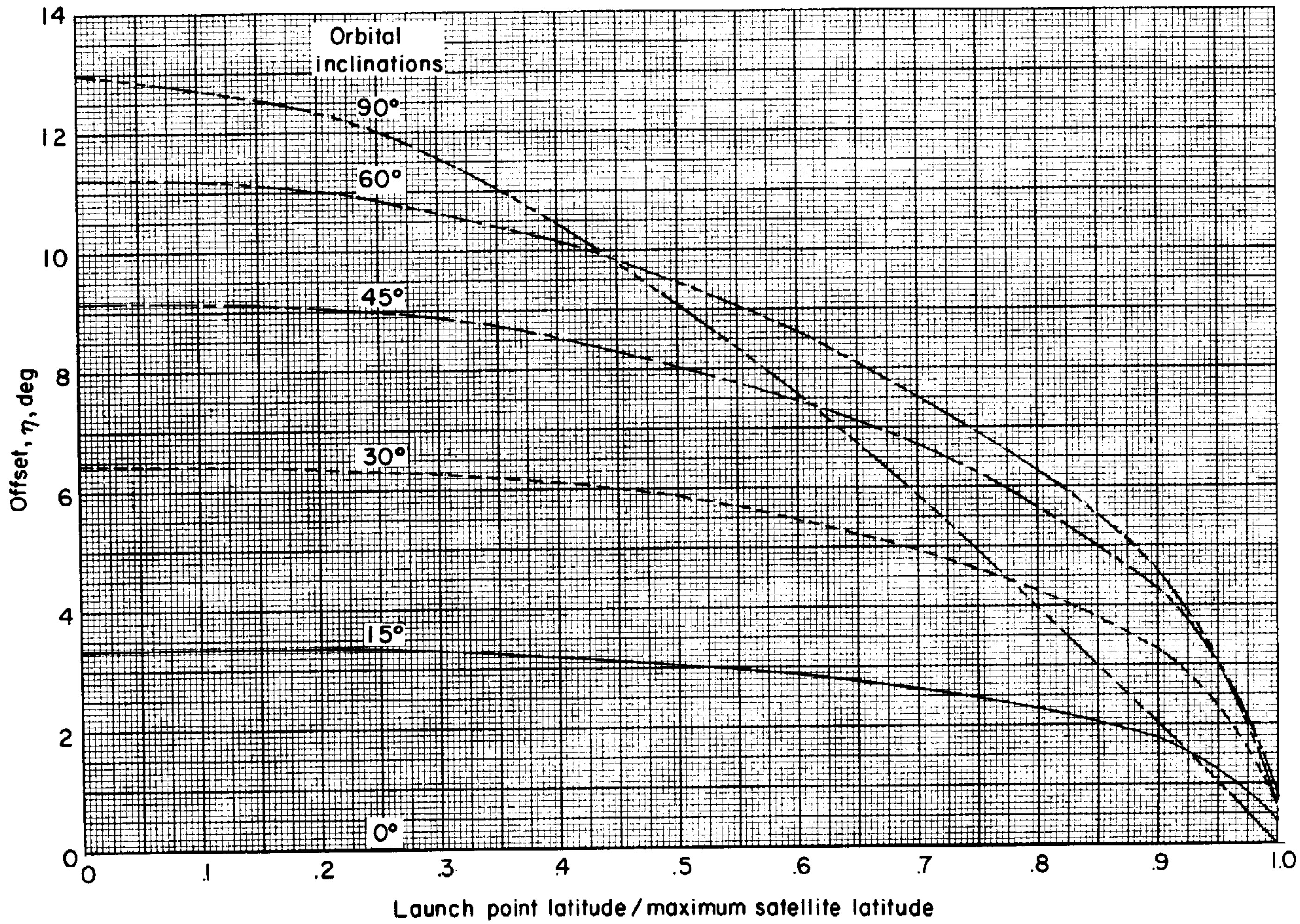


Figure 2.- Geometry of offset due to delay time.



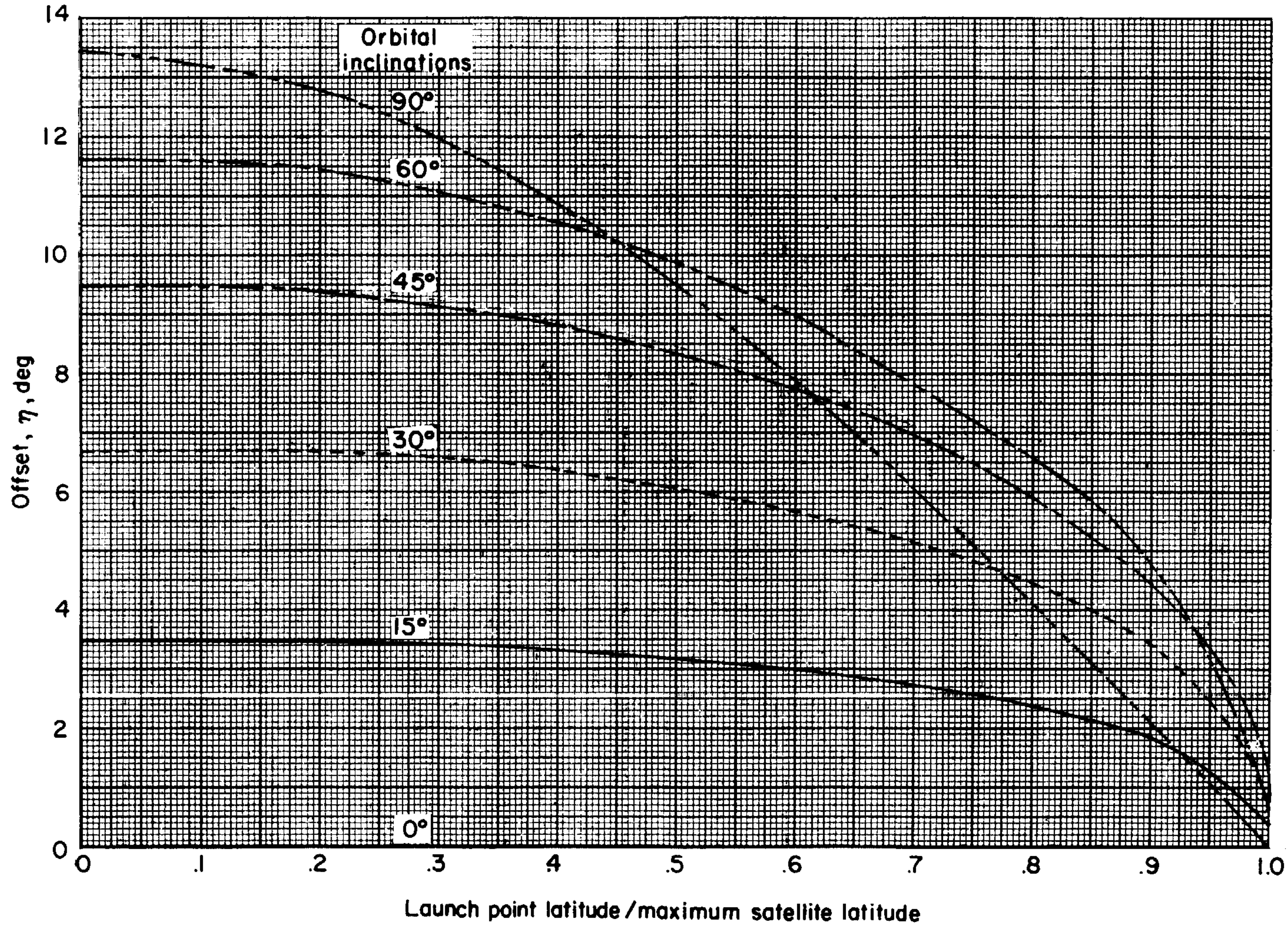
(a) 400-nautical-mile satellite.

Figure 3.- Required offset for once-a-day rendezvous as a function of the ratio of launch-point latitude/maximum satellite latitude for orbital inclinations of 15°, 30°, 45°, 60°, and 90°.



(b) 500-nautical-mile satellite.

Figure 3.- Continued.



(c) 600-nautical-mile satellite.

Figure 3.- Concluded.

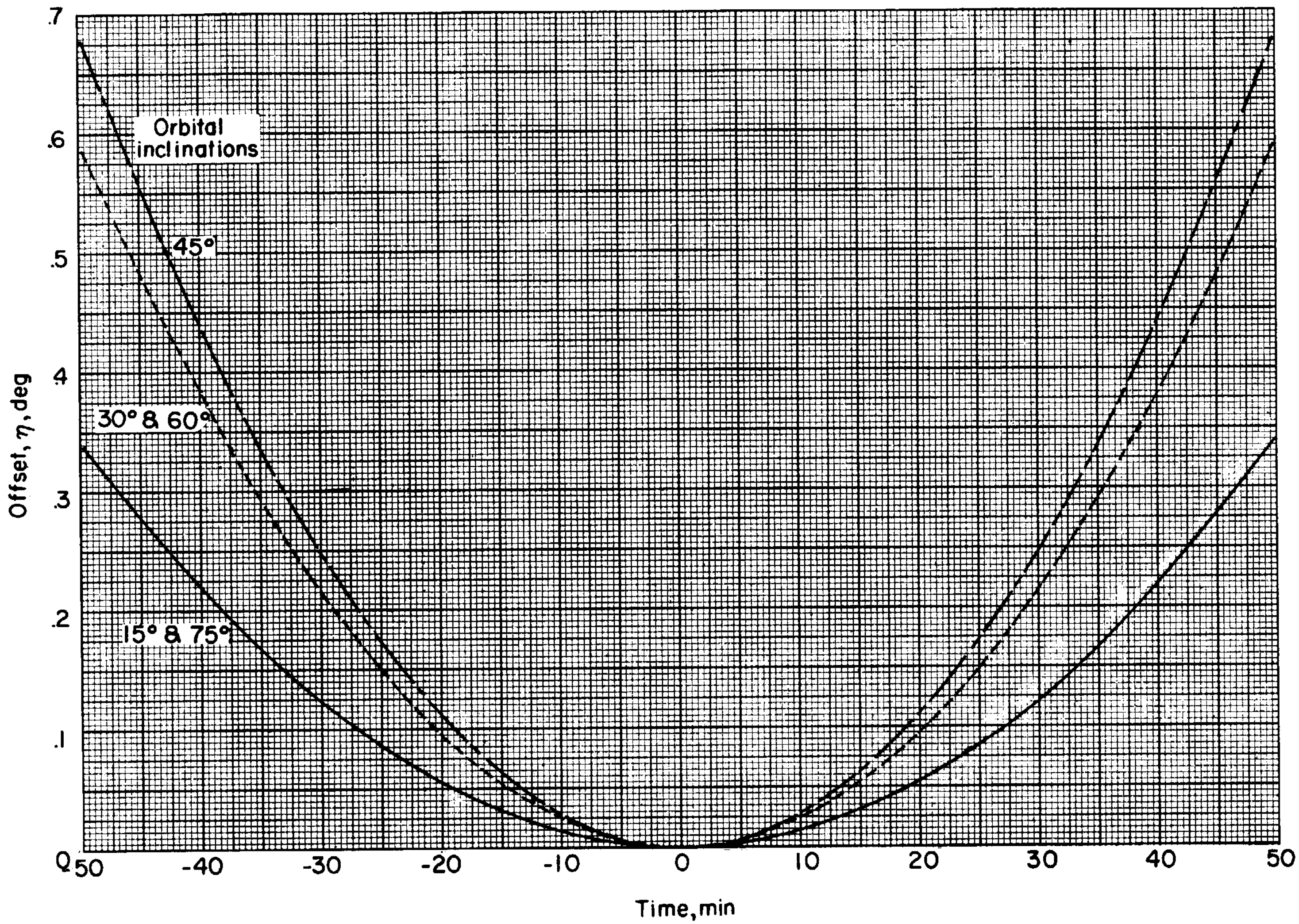
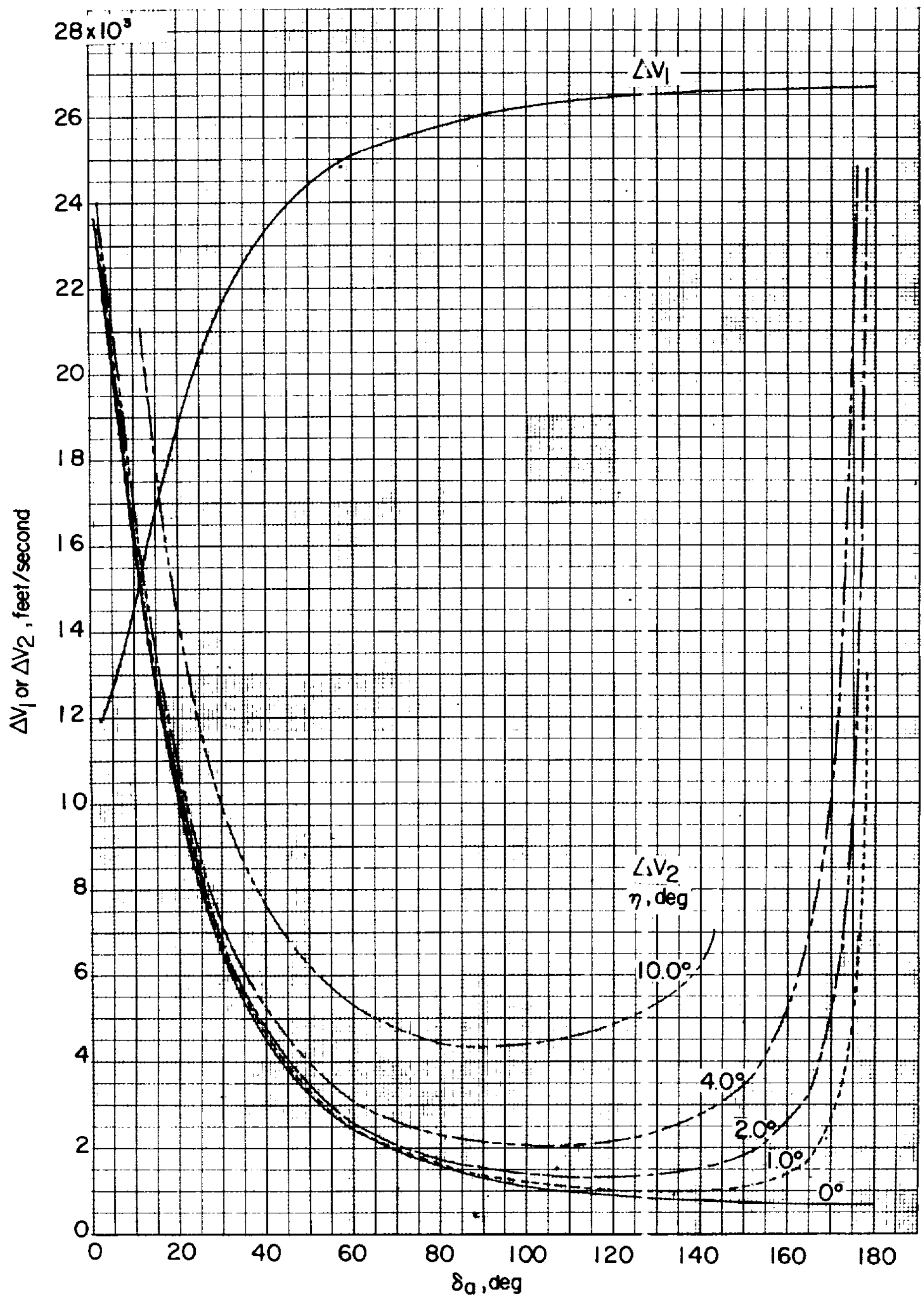


Figure 4.- Offset near the tangency condition as a function of time for orbital inclinations of 15°, 30°, 45°, 60°, and 75° for 400 nautical miles.



(a) 400-nautical-mile satellite.

Figure 5.- The impulsive velocity increments at launch and apogee required for rendezvous as a function of angular distances to apogee (that is, rendezvous). Offsets of 0°, 1°, 2°, 4°, and 10°.

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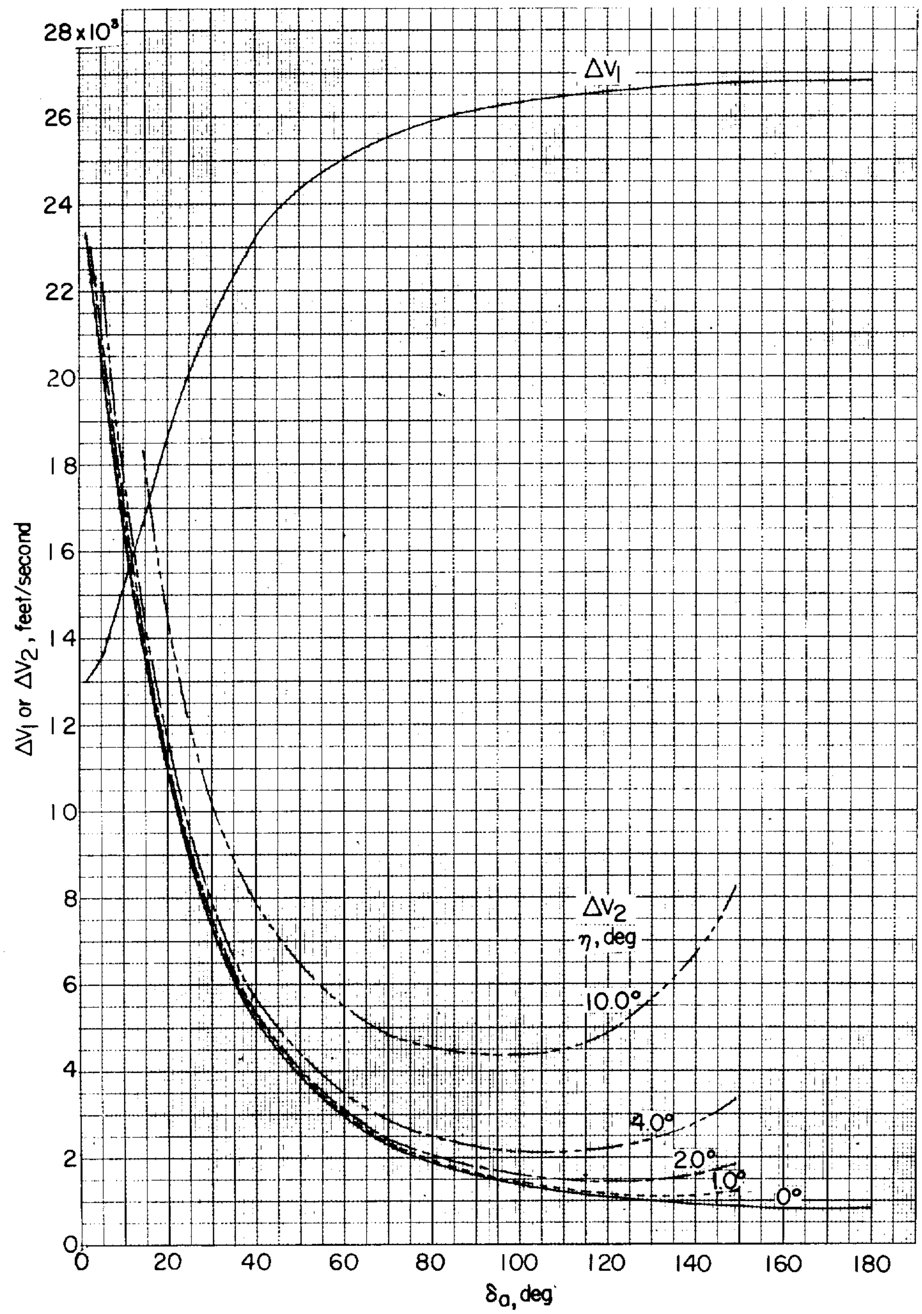
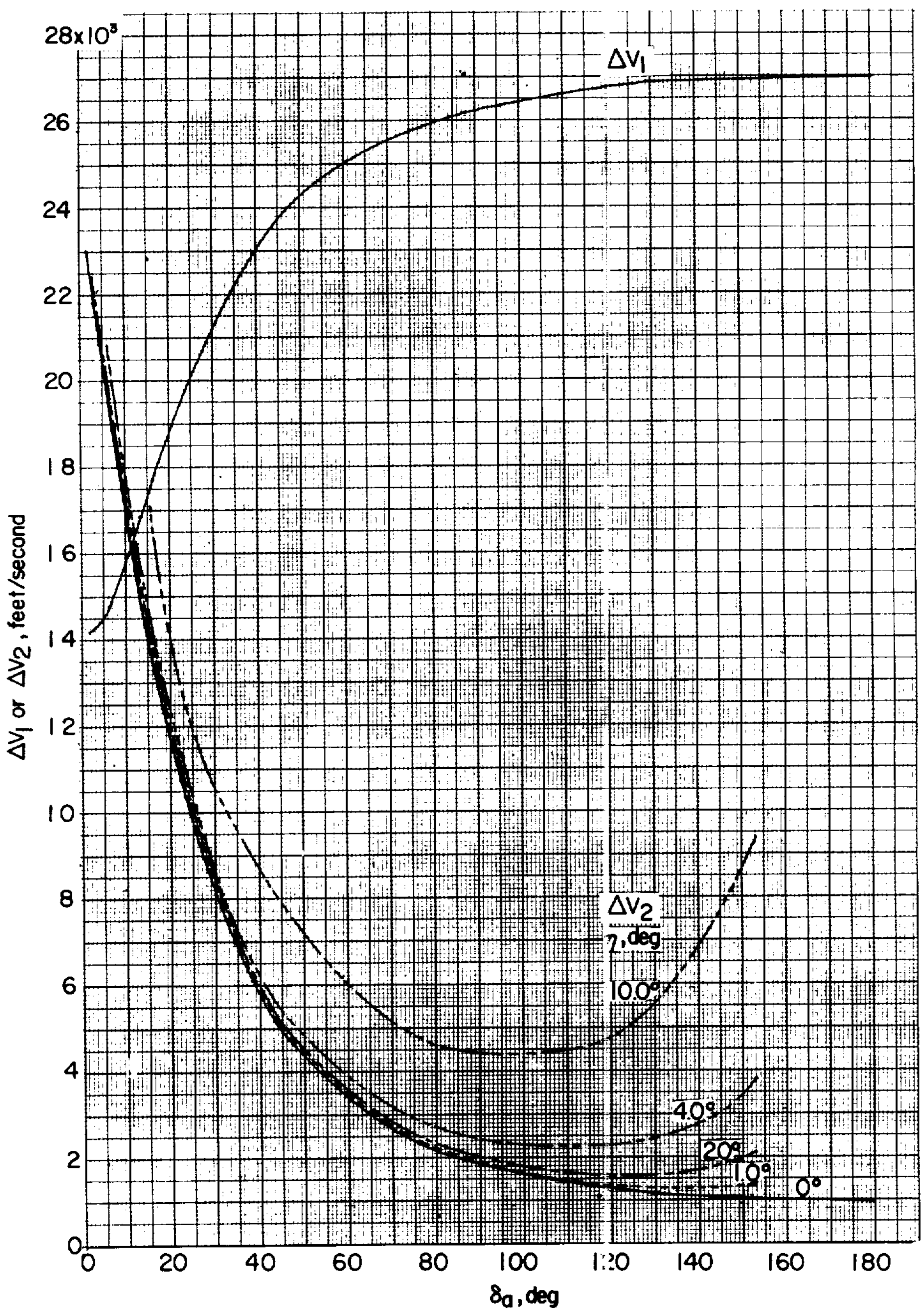


(b) 500-nautical-mile satellite.

Figure 5.- Continued.



(c) 600-nautical-mile satellite.

Figure 5.- Concluded.

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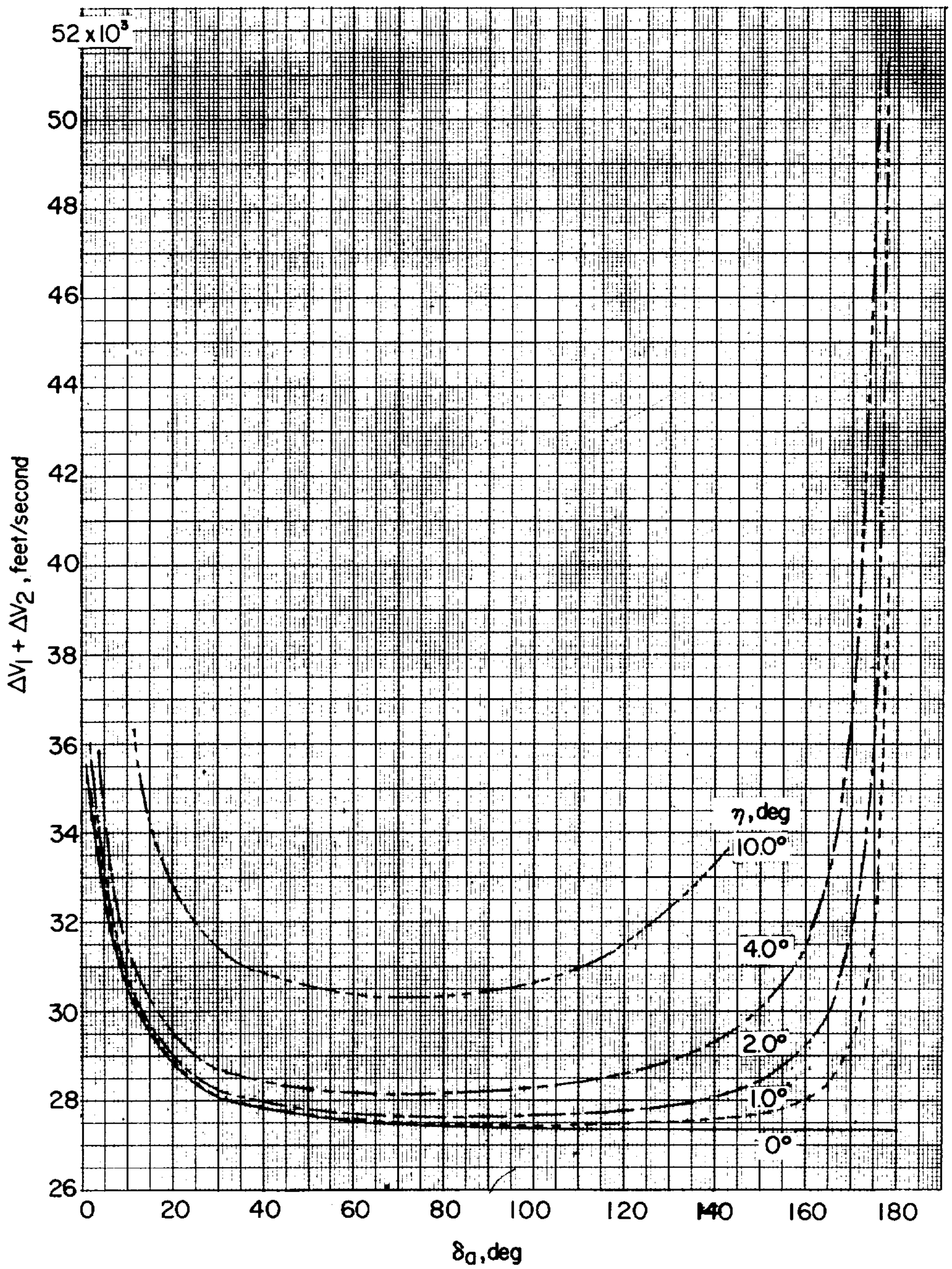
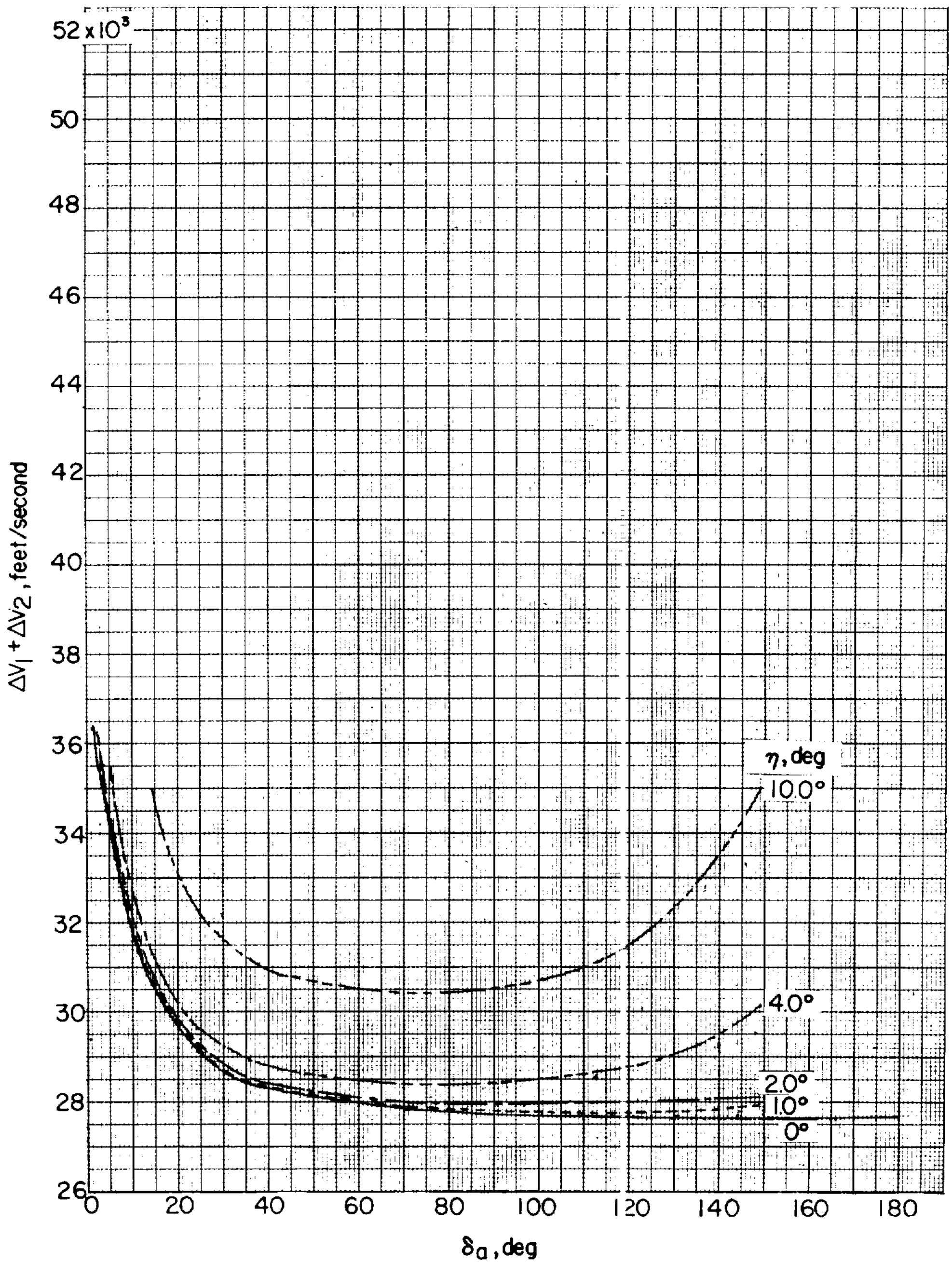


(a) 400-nautical-mile satellite.

Figure 6.- The total impulsive velocity increment required for rendezvous as a function of angular distance to apogee (that is, rendezvous).



(b) 500-nautical-mile satellite.

Figure 6.- Continued.

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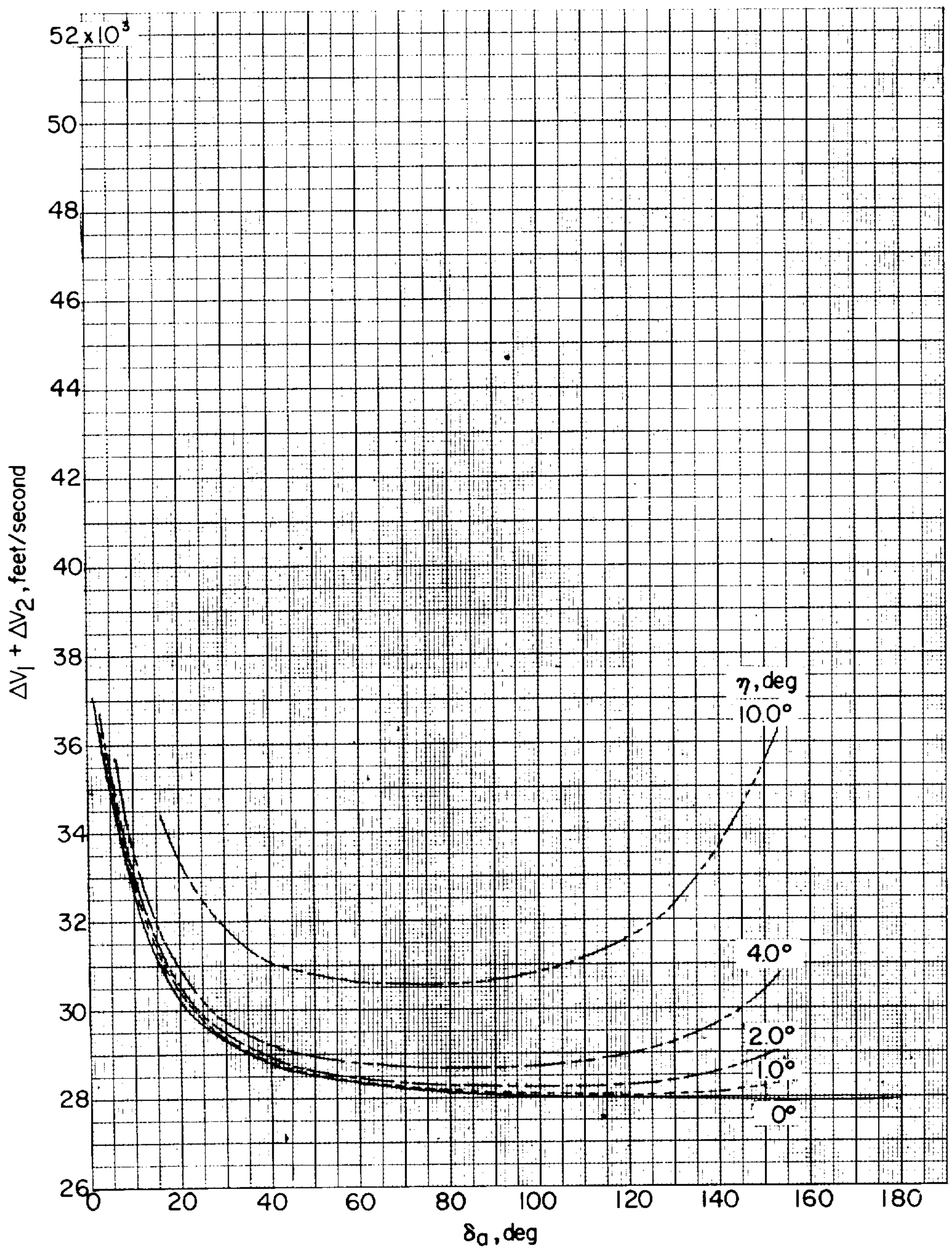
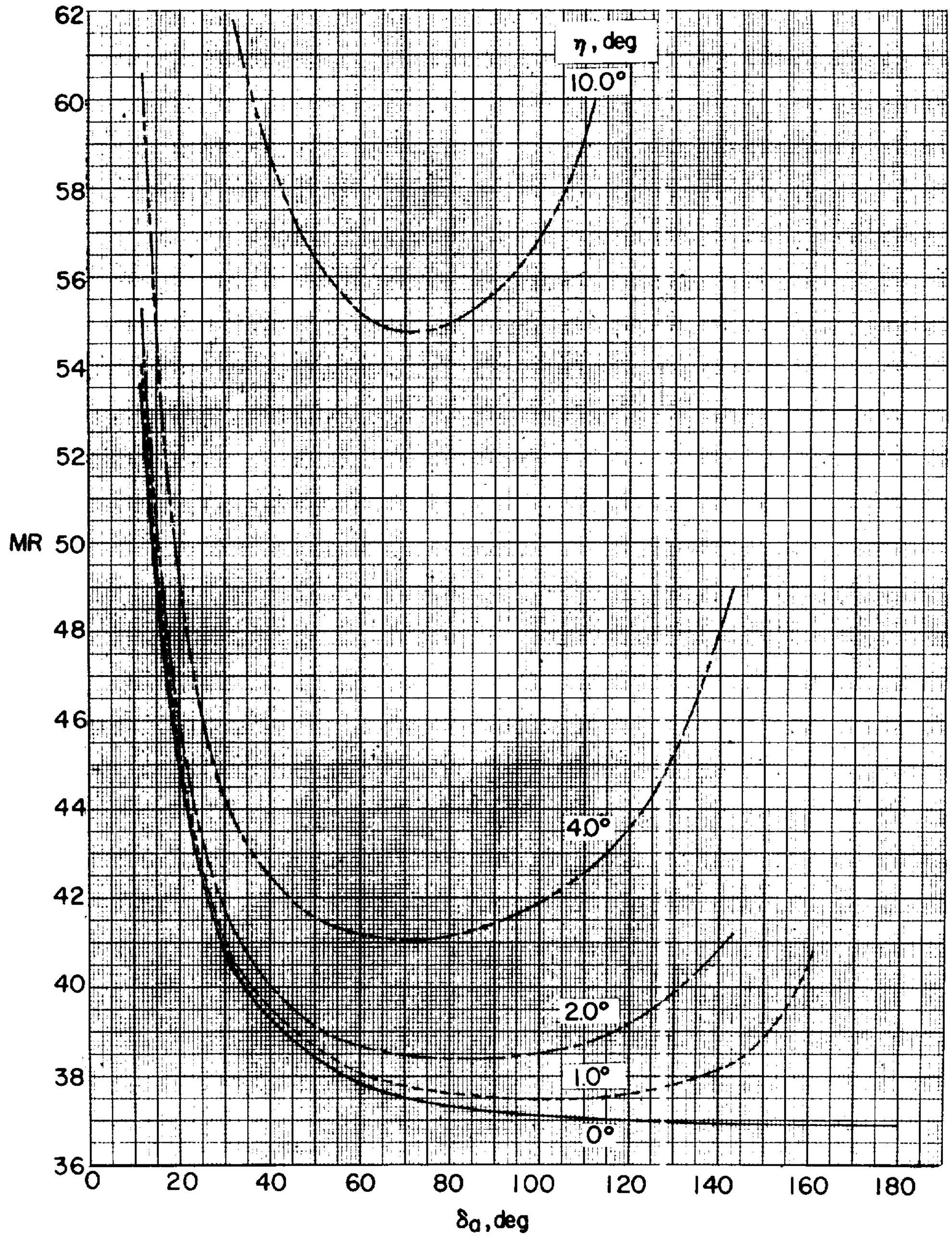


(c) 600-nautical-mile satellite.

Figure 6.- Concluded.



(a) 400-nautical-mile satellite.

Figure 7.- The mass ratio required for rendezvous as a function of angular distance to apogee (that is, rendezvous) for various amounts of offset of the launch point from the orbital plane. I_{sp} = 235 sec.

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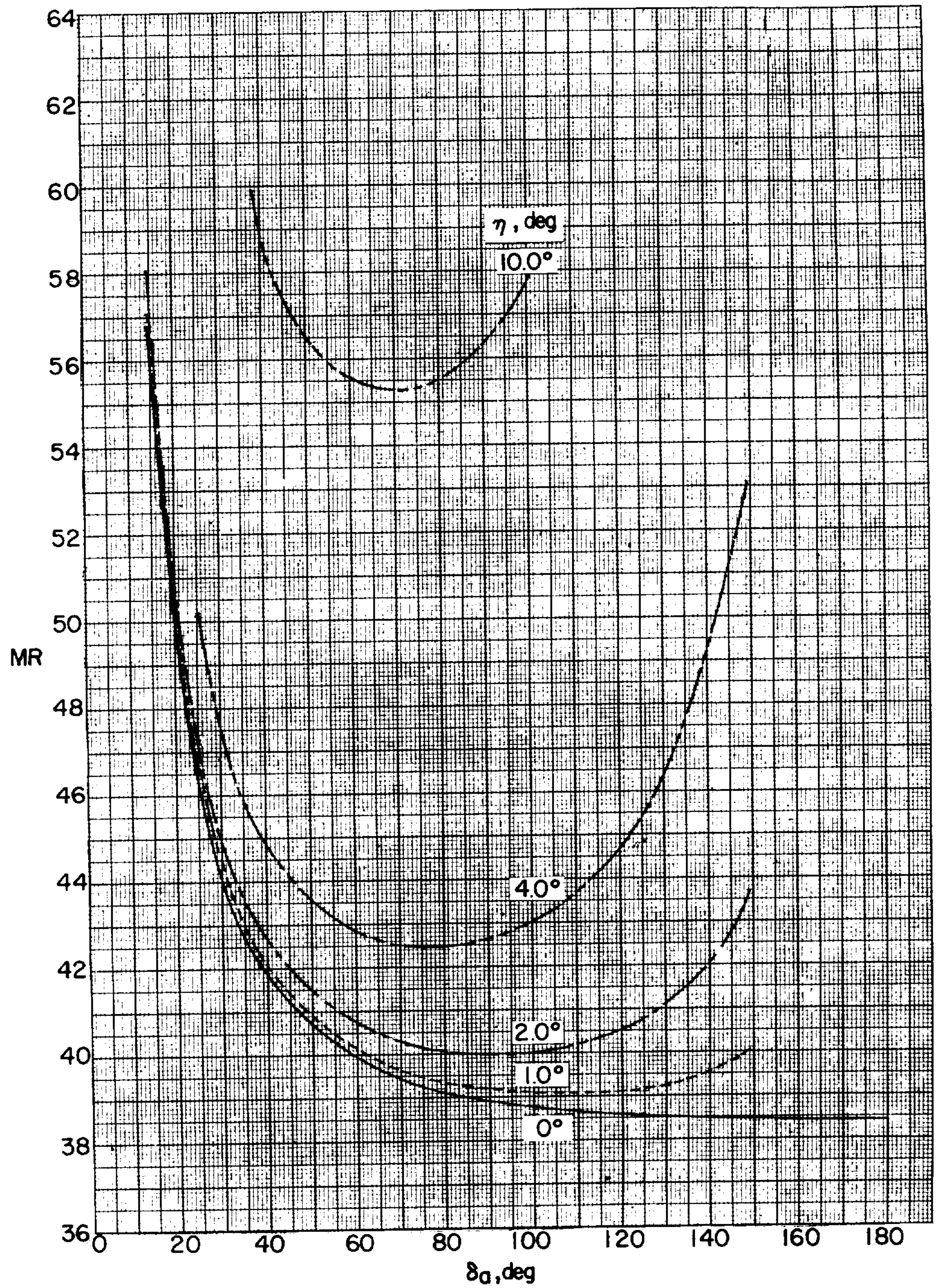
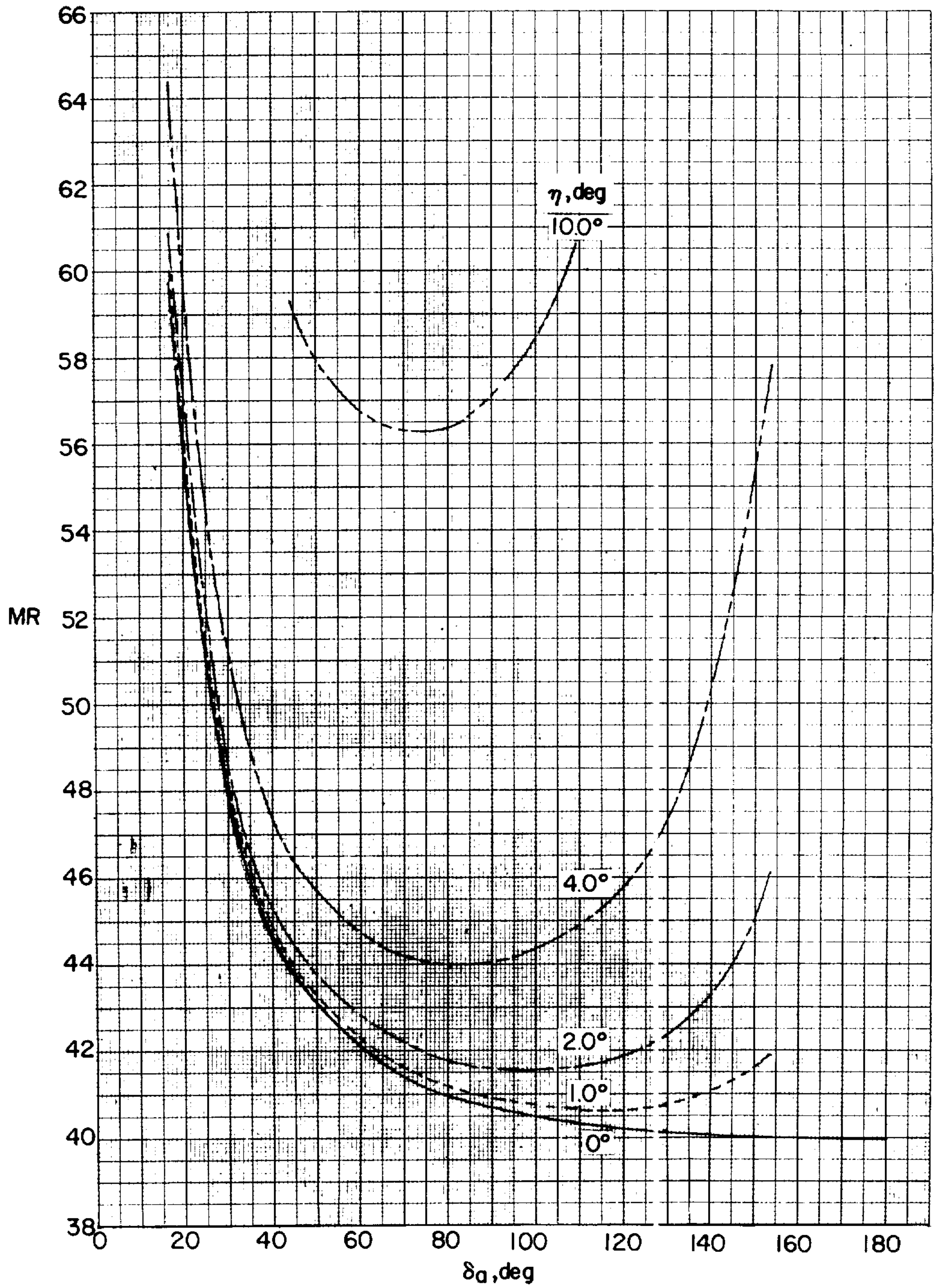


(b) 500-nautical-mile satellite.

Figure 7.- Continued.



(c) 600-nautical-mile satellite.

Figure 7.- Concluded.

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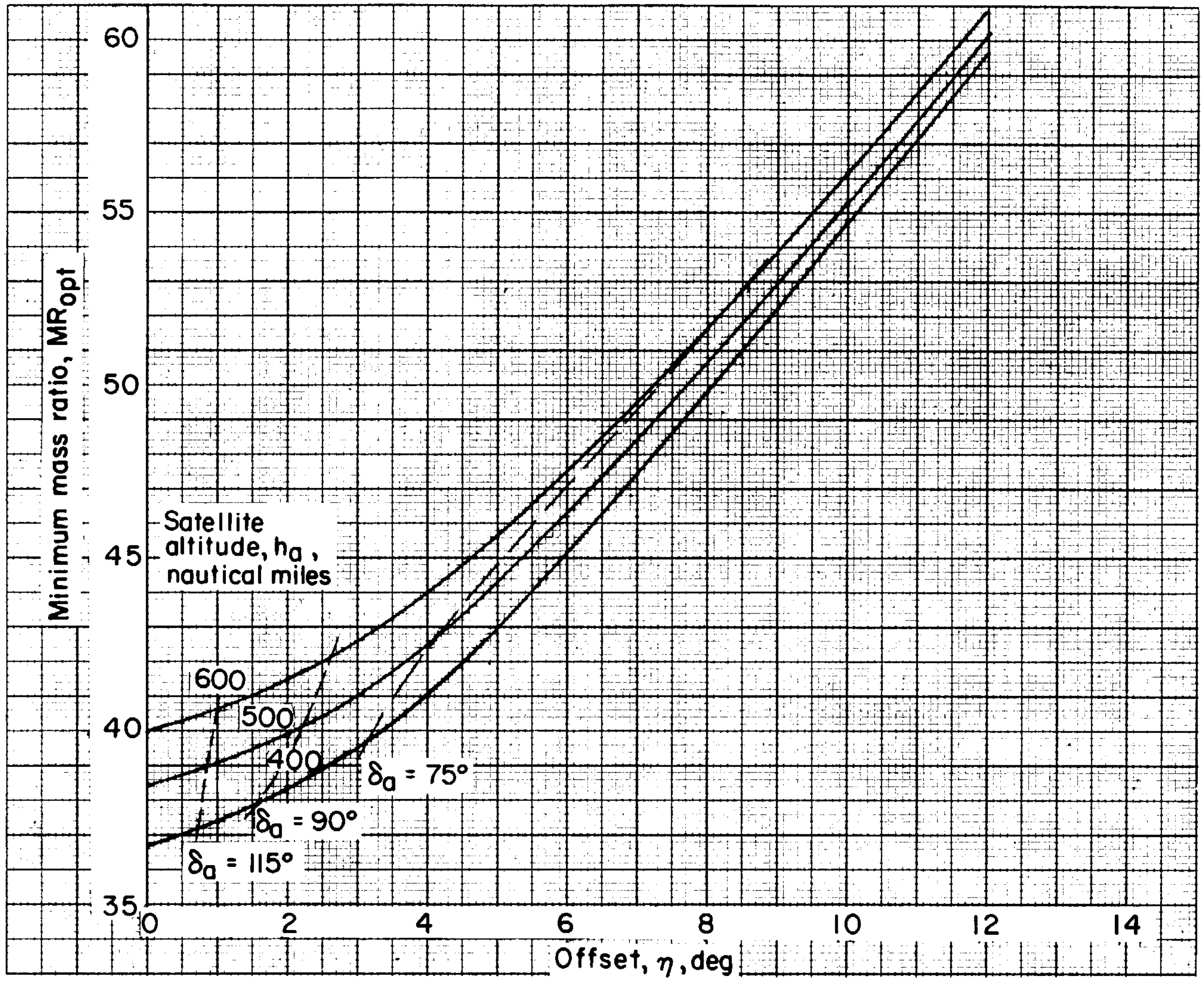
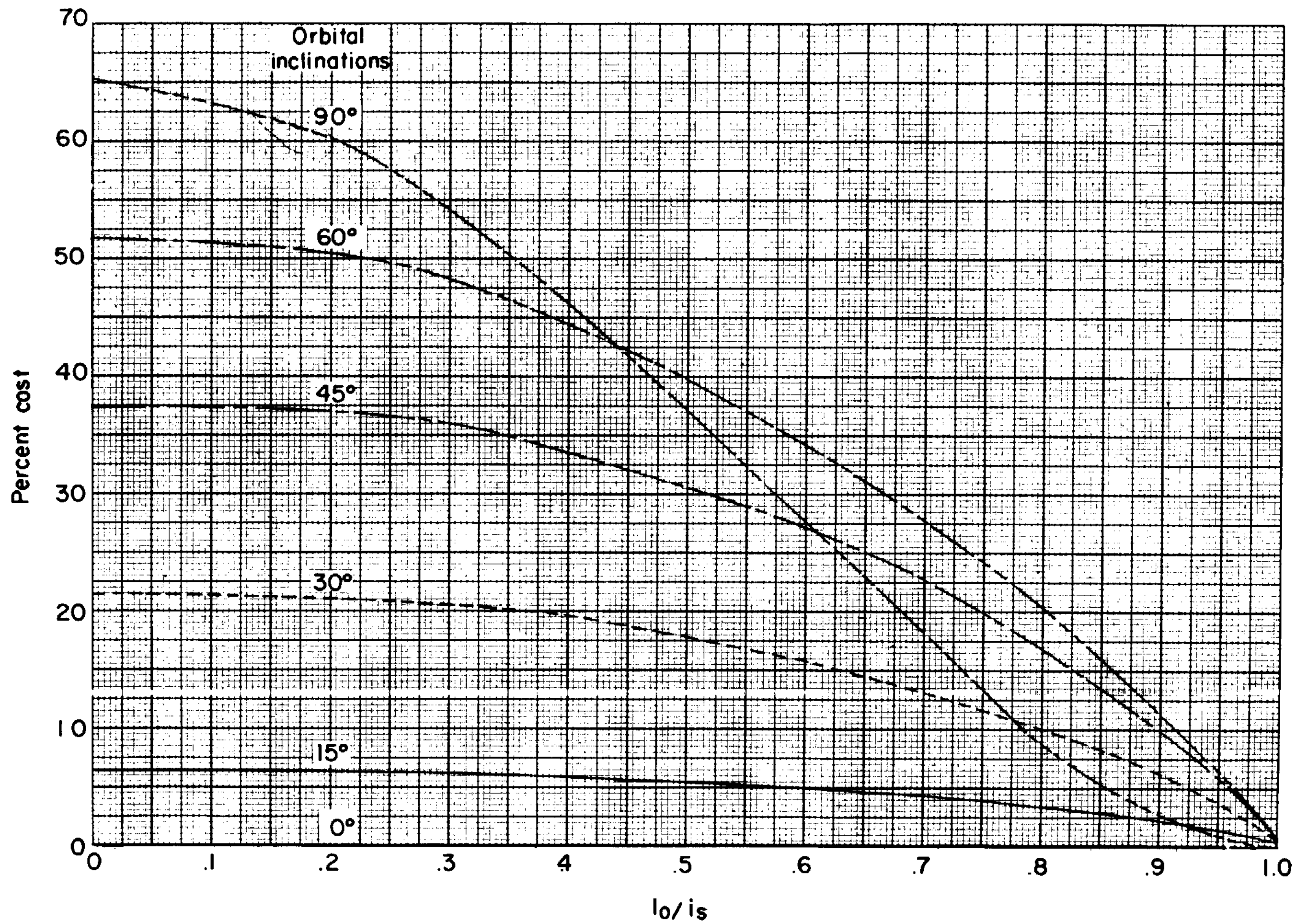
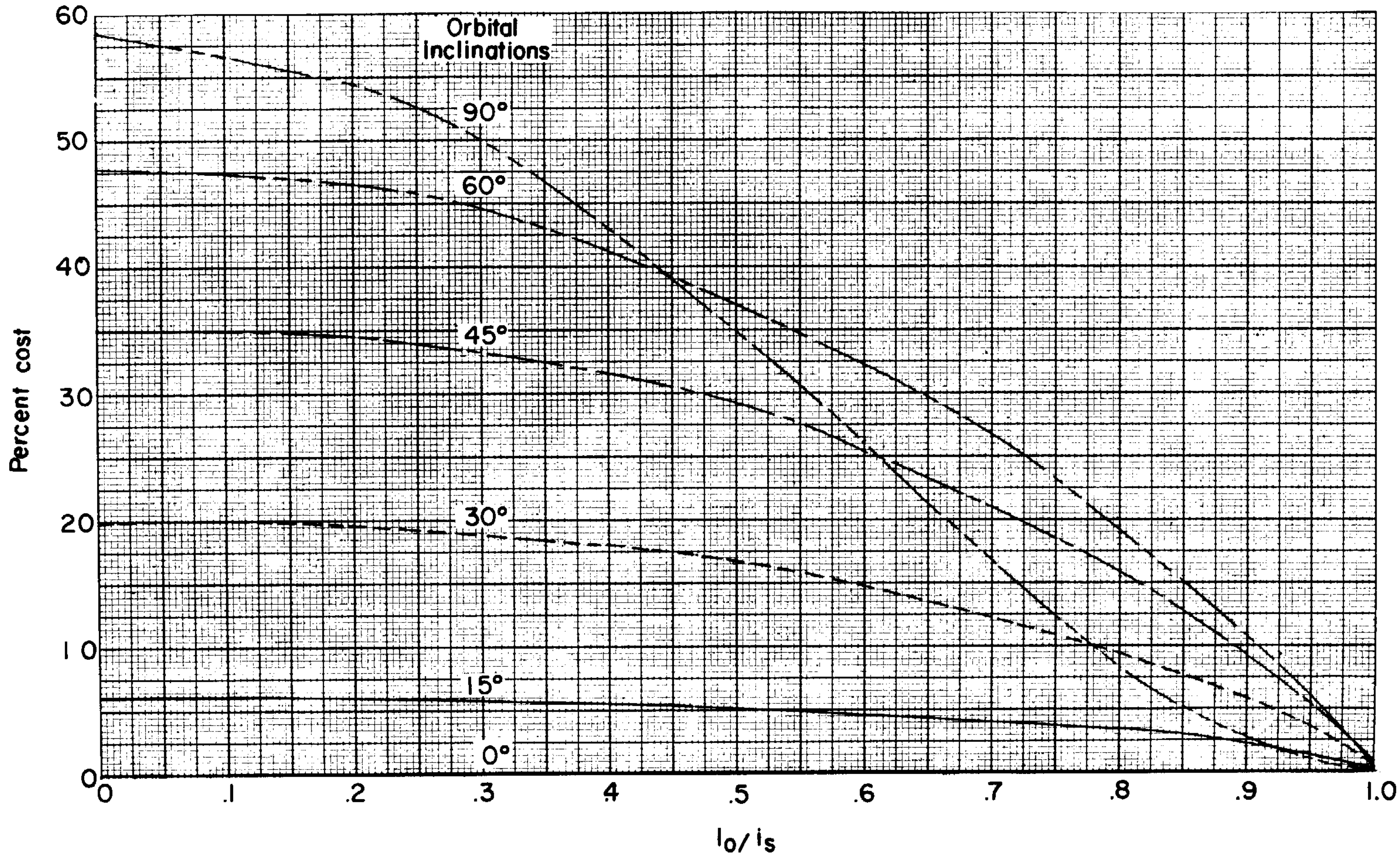


Figure 8.- Minimum mass ratio for launch to rendezvous as a function of the offset of the launch point from the orbital plane for three satellite altitudes. I_{sp} = 235 sec.



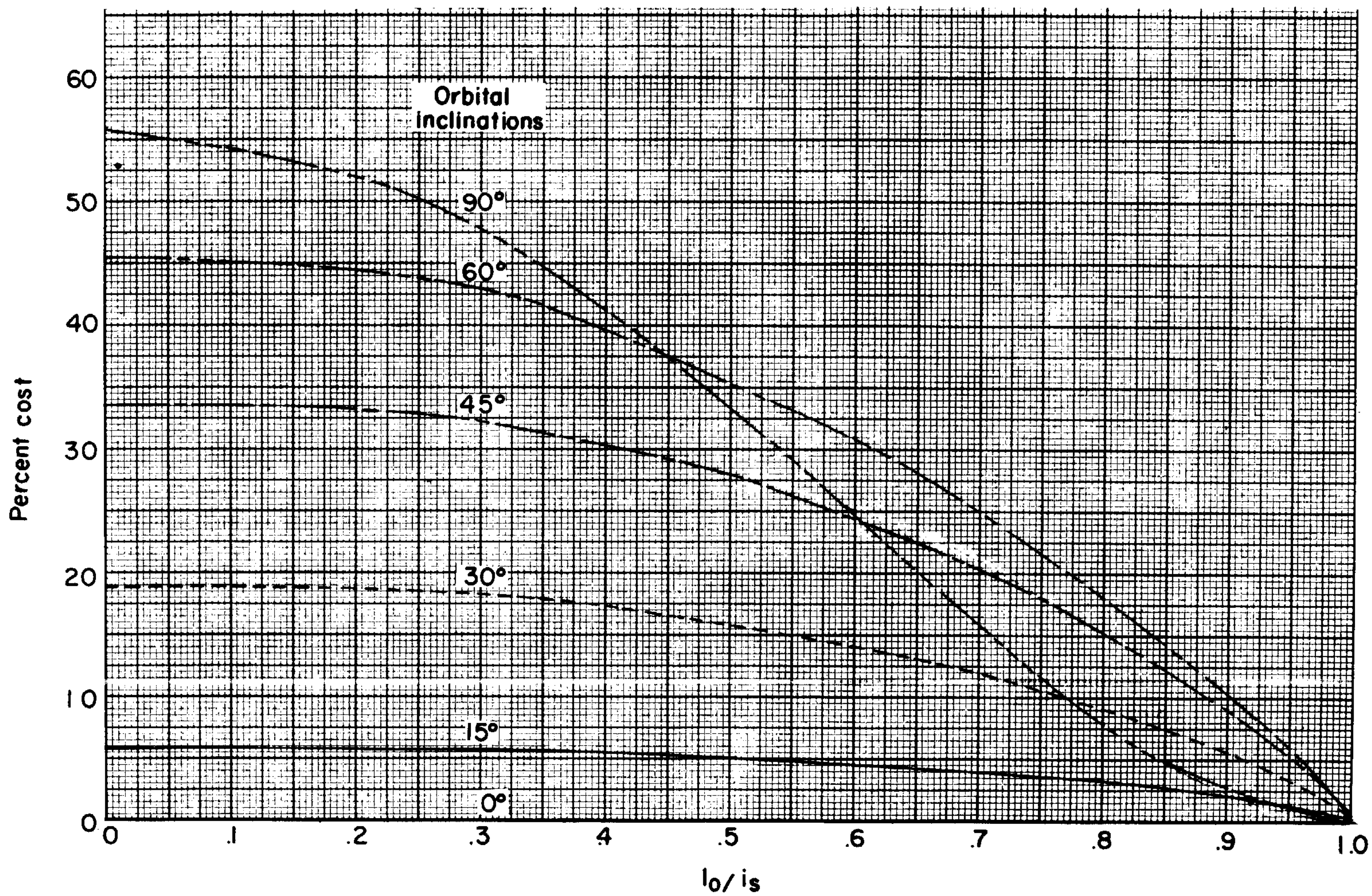
(a) 400-nautical-mile satellite.

Figure 9.- The minimum required cost of once-a-day rendezvous as a function of launch-point latitude. The cost is given as the percent increase in mass ratio above that required for an inplane launch. I_{sp} = 235 sec.



(b) 500-nautical-mile satellite.

Figure 9.- Continued.



(c) 600-nautical-mile satellite.

Figure 9.- Concluded.

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